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On the shear stress parameter of thin-walled tubular specimens under torsional loading

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Abstract

In the most of existing fatigue life prediction models, the maximum shear stress on a tubular specimen's outer surface is chosen as the shear stress parameter to predict fatigue lifetime for torsional loading. It has been experimentally found that the fatigue lifetime observed under fully reversed torsion is always higher than that under tension-compression for the same equivalent stress level. In this paper, an improved shear stress fatigue damage parameter based on Buch's two-parameter model was proposed. In this parameter, the influences of both shear stress gradient and cyclic strain hardening were taken into account. A serial of tensile-compressive and torsional loading tests were carried out on 2024-T4 aluminum alloy to validate the new parameter. Four σ_{eq}-N_f curves were plotted using different shear stress parameters. Compared with other shear stress parameters, the σ_{eq}-N_f curve translated from the new parameter made best coincide with the tension-compression *S-N* curve.

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Keywords: Multiaxial fatigue; Shear stress gradient; Supporting effect; Cyclic hardening

Nomenclature

F_a	Axial force amplitude
M_a	Torsional moment amplitude
D	Outer diameter
d	Inner diameter

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σ_{eq}	von Mises equivalent stress
R	Outside radius
τ_{max}	Shear stress on the outer surface
A	Material constant
h	Material constant
ε'	Residual strain

1. Introduction

Engineering components and structures often undergo multiaxial loading. These products designed by direct application of uniaxial fatigue theory cannot meet fatigue strength design requirements generally. For this reason, the problem of multiaxial fatigue assessment has long been investigated and continues to be investigated by many researchers.

One propose of the multiaxial fatigue study is to find a model which can be used to estimate the multiaxial fatigue life by using only uniaxial fatigue data. Most existing models need uniaxial fatigue data obtained from both fully reversed tension-compression and torsion tests [1-4]. In the fully reversed torsion tests, thin-walled tubular specimens are commonly used. The specimen's wall thickness should be large enough to avoid instabilities during cyclic loading without violating the thin-walled tube criterion, i.e. the ratio of average diameter to wall thickness should be 10:1 or larger [5]. At present, the maximum shear stress on the outer surface is the most commonly used parameter to represent the fatigue damage [6-10]. In addition, the shear stress on the surface of mean diameter has also been used [11, 12]. There is no reasonable final conclusion on the selection of shear stress parameter until now.

It has already been experimentally indicated that, for a certain material, equivalent (von Mises) stress-lifetime curves, which are respectively transformed from S-N curve and τ-N curve, do not coincide with each other. In order to eliminate this discrepancy, an improved shear stress fatigue damage parameter based on Buch's two-parameter model was proposed. Meanwhile, a series of multiaxial fatigue tests were conducted to validate the new parameter. Owing to its physical basis, this parameter is particularly suitable for the representation of specimen's fatigue damage condition and the lifetime prediction for torsional loading.

2. Test programs

2.1. Material and Specimen

2024-T4 was used in this study. The geometry of the specimen is shown in Figure 1. The specimen was manufactured from a bar 30mm in diameter of aluminium alloy, 2024-T4. The material properties at the room temperature are shown in Table 1. Eleven specimens ware used for tensile-compressive fatigue tests and 13 specimens for torsional fatigue tests on MTS809 electro-hydraulic servo fatigue testing machine.

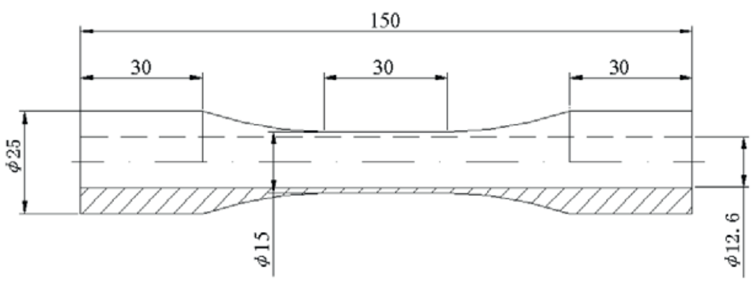

Figure 1 the geometry of the specimen (mm)

Table 1 Mechanical properties of 2024-T4

E/GPa	σ_y /MPa	σ_u /MPa	ν	σ_f /MPa
73	400	545	0.33	643.44
ε_f	K/MPa	n	K' /MPa	n'
0.18	850	0.158	870	0.097

The results for the 24 specimens are given in Table 2. The tensile-compressive stress and shear stress were respectively calculated by

$$\sigma_a = \frac{4F_a}{\pi(D^2 - d^2)} \tag{1}$$

$$\tau_a = \frac{16M_a D}{\pi(D^4 - d^4)} \tag{2}$$

where F_a and M_a are axial force amplitude and torsional moment amplitude respectively, D and d are outer diameter and inner diameter of the gauge section.

The test results are listed in Table 2. In Table 2, σ_{eq} stands for von Mises equivalent stresses. For tension-compression, $\sigma_{eq} = \sigma$ and for torsion, $\sigma_{eq} = \sqrt{3}\ \tau$.

Table 2 Fatigue life under tensile-compressive and torsion loading

Load case	σ_{eq} /MPa	Specimen No.	N_f	N_{50}
Tension - Compression	200	AT-43	335621	297359
		AT-44	263460	
		AT-49	182157	
	250	AT-10	34519	56316
		AT-39	64118	
		AT-40	80695	
	300	AT-67	11312	12180
		AT-68	13115	
	336.68	AT-02	6693	6873
		AT-03	6868	
		AT-04	7066	
Torsion	192.76	AT-72	2058844	1689182
		AT-88	—*	
		AT-92	1385893	
	240.95	AT-78	—*	293646
		AT-81	306684	
		AT-80	281163	

Load case	σ_{eq} /MPa	Specimen No.	N_f	N_{50}
	289.14	AT-70	—*	49912
		AT-71	42031	
		AT-76	59271	
	350.00	AT-93	10836	8911
		AT-94	7277	
		AT-95	7916	
		AT-96	10101	

(* Fatigue crack didn't occur in the test section)

3. Novel shear stress parameter

It has been experimentally found that the fatigue lifetime observed under fully reversed bending are always higher than that under tension-compression for the same stress level [13]. This beneficial influence of the stress gradient is called "supporting effect". At present, many approaches have already been established to modelling the supporting effect under uniaxial normal cyclic stress. Among these approaches, Buch's two-parameter model [14] has both simplification and effectivity.

3.1. Buch's two-parameter model

In Buch's two-parameter model, the supporting effect of stress gradient is taken into account. It is concluded that a fatigue crack initiates only if the stress (σ_K), which is acquired on the material layer of critical depth h, exceeds a critical value $A\sigma_d$. σ_d is the fatigue strength at the same fatigue lifetime corresponding to the load with no stress gradient, e.g. reserved tension-compression . Material constants h and A are related to the types of materials and loadings, and determined experimentally.

For a tubular specimen subjected to torsion loading, the theoretical shear stress gradient is, if the material is in linear elastic state,

$$\frac{d\tau}{dr} = \frac{\tau_{max}}{R} \tag{1}$$

where R is the outside radius, τ_{max} is the shear stress on the outer surface which can be calculated as follow

$$\tau_{max} = \frac{M}{W} \tag{2}$$

where M is the amplitude of torsional moment , W is section modulus in torsion.

As previously stated, the stress on a material layer of critical depth h is denoted as τ_K , so the shear stress gradient can be also expressed as

$$\frac{d\tau}{dr} = \frac{\Delta\tau}{\Delta r} = \frac{\tau_{max} - \tau_K}{h} \tag{3}$$

Therefore,

$$\tau_{\rm K} = \tau_{\max} - \frac{{\rm d}\tau}{{\rm d}r} h = \tau_{\max} - \frac{\tau_{\max}}{R} h = \left(1 - \frac{h}{R}\right)\tau_{\max} \tag{4}$$

Thus, the shear stress parameter is

$$\tau_{\rm d} = \frac{\tau_{\rm k}}{A} = \frac{1}{A}\left(1 - \frac{h}{R}\right)\tau_{\max} \tag{5}$$

where h and A are material constants, $\tau_{\max}$ is the shear stress on the outer surface, R is the outside radius.

3.2. The effect of cyclic strain hardening or softening

Most of materials show cyclic strain hardening or softening under cyclic loading. The behavior of cyclic hardening or softening leads to the discrepancy between monotonous and cyclic stress-strain curve. Although material data about cyclic stress-strain curve under torsional loading is limited, data under tensile-compressive loading is available. Since the shapes of the hysteresis loops are quite similar for these two kinds of loading cases, it can be assumed that cyclic stress-strain curves are the same for torsional and tensile-compressive loading. The cyclic stress-strain curve and monotonic stress-strain curve for 2024-T4 is shown in Figure 2.

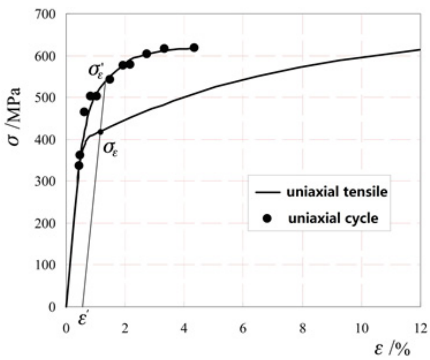

Figure 2 cyclic stress-strain curve and monotonic stress-strain curve for 2024-T4

In Figure 2, it can be observed that the discrepancy between these two kinds of stress-strain curves various with the magnitude of load. This means that the degree of cyclic hardness various with the magnitude of load. To take this phenomenon into account, a variational parameter A, which is modified by the magnitude of loading, is proposed as follow,

$$A = A_0 \frac{\sigma_{\rm eq\,C}}{\sigma_{\rm eq\,M}} \tag{6}$$

where A_0 is the reference value which is set to be 1.05 for 2024-T4 aluminum alloy[15], σ'_ε is the stress correspondent to the residual strain ε' in the cyclic stress-strain curve, and σ_ε is the stress correspondent to the residual strain ε' in the monotonic stress-strain curve. In detail, the factor $\sigma_{\rm eq\,C} / \sigma_{\rm eq\,M}$, which is related to the load magnitude, can be solved by following equations:

$$\varepsilon' = \varepsilon_{\mathrm{C}} - \frac{\sigma_{\mathrm{eq\,C}}}{E} = \varepsilon_{\mathrm{M}} - \frac{\sigma_{\mathrm{eq\,M}}}{E} \tag{7}$$

$$\varepsilon_{\mathrm{C}} = \frac{\sigma_{\mathrm{eq\,C}}}{E} + \left(\frac{\sigma_{\mathrm{eq\,C}}}{K'}\right)^{\frac{1}{n'}} \tag{8}$$

$$\varepsilon_{\mathrm{M}} = \frac{\sigma_{\mathrm{eq\,M}}}{E} + \left(\frac{\sigma_{\mathrm{eq\,M}}}{K}\right)^{\frac{1}{n}} \tag{9}$$

where ε' is the residual strain, $\sigma_{\mathrm{eq\,M}}$ and $\sigma_{\mathrm{eq\,C}}$ are von Mises equivalent stresses for monotonic and cyclic stress-strain curve, ε_{M} and ε_{C} are the strains which are relevant to $\sigma_{\mathrm{eq\,M}}$ and $\sigma_{\mathrm{eq\,C}}$ on the stress-strain curves. For a certain torsional loading, $\sigma_{\mathrm{eq\,C}}$ is set to be $\sqrt{3}\tau_{\max}$. Then, lining Eq.(9), Eq. (10) and Eq.(11), $\sigma_{\mathrm{eq\,M}}$ can be calculated.

The discussion in Section 3.1 is restricted in linear elastic condition. With the increase of loading, the specimen's section enters plastic deformation gradually from outside to inside. However, owing to the newly proposed parameter *A*, the cyclic plasticity can be taken into account. So, in elastic-plastic state, the shear stress parameter τ_{d} can still be obtained by Eq. (7).

3.3. Determination of parameter h

In the original Buch's two-parameter model, the parameter *h* is determined by data-fitting. This method is tedious and lack of operability. According to its essential conception, it can be found that there are some similarities between Buch's two-parameter model and Stress Field Intensity Approach (SFI approach). They both take a characteristic stress of a certain area as a criterion to estimate the failure of the material. The sizes of the area are both material constants. In SFI approach, this is called field-dimension and has been defined clearly. It is regarded as the inherent attribute of a material and does not vary with the loading. The field-dimension is determined by fitting the experimental data. Considering of the description of Buch's two-parameter model, the critical depth *h* doesn't change with the amplitude of loading either. This is similar to the field-dimension of SFI approach. So, a material's field-dimension is used as the critical depth *h* in this paper. For 2024-T4, the field-dimension is 0.185mm [16].

4. Validation of the proposed parameter

Before the validation, relevant factors, such as $\sigma_{\mathrm{eq\,C}} / \sigma_{\mathrm{eq\,M}}$ and *A*, should be determined first. Detailed values of these factors are listed in Table 3. It can be seen from the table that, the parameter *A*, which is taking the cyclic strain hardening into account, indeed varies with the loading amplitude. Then, according to Eq.(7), the newly proposed shear stress parameter τ_{d} is recalculated. In Table 3, $\sigma_{\mathrm{eq\,d}}$ is the von Mises equivalent stress calculated by τ_{d} .

Table 3 Detailed values for the shear stress parameter calculation

$\tau_{\max}$ /MPa	ε'	$\sigma_{\mathrm{eq\,C}} / \sigma_{\mathrm{eq\,M}}$	A	τ_{d} /MPa	$\sigma_{\mathrm{eq\,d}}$ /MPa
192.76	0.00005	1.87	1.96	95.79	165.90
240.95	0.00051	1.62	1.71	137.77	238.62
289.14	0.00337	1.45	1.52	185.41	321.13

τ_{max} /MPa	ε'	$\sigma_{eq\,C}$ / $\sigma_{eq\,M}$	A	τ_d /MPa	$\sigma_{eq\,d}$ /MPa
350	0.02412	1.28	1.35	253.08	438.34

Then, four σ_{eq}-N curves are plotted in Figure 3. Among these curves:

- "Tensile-Compression" stands for the tensile-compressive *S-N* curve.
- "Torsion (Out)" stands for the equivalent stress-lifetime curve, of which the von Mises stress σ_{eq} is calculated by the shear stress parameter τ_{max} which is obtained on the outer surface. The shear stress parameter τ_{max} is common used in the researches to represent the fatigue damage condition.
- "Torsion (Midsection)" stands for the equivalent stress-lifetime curve of which the von Mises stress σ_{eq} is calculated by the shear stress parameter τ_{mid} which is obtained on the midsection surface. According to Ref. [12], τ_{mid} can be given by

$$\tau_{mid} = \frac{M_a}{r_m A} \tag{10}$$

where M_a is the amplitude of torsional moment, A is specimen cross-section area, and r_m is midsection radius.

- "Torsion (Proposed)" stands for the equivalent stress-lifetime curve of which the von Mises stress σ_{eq} is calculated by the newly proposed shear stress parameter τ_d .

In Figure 3, it can be observed that the $\sigma_{eq} - N_f$ curve translated from common used τ_{max} is largely different from the tensile-compressive *S-N* curve at the same fatigue lifetime. The $\sigma_{eq} - N_f$ curve translated from τ_{mid} is closer to the *S-N* curve. To some extent, this also indicates that it is inappropriate to regard the shear stress on the outer surface as the damage parameter for fatigue lifetime prediction, without considering the influence of shear stress gradient. By contrast, the $\sigma_{eq} - N_f$ curve translated from τ_d makes best coincide with the *S-N* curve in tension-compression, owing to its consideration of both shear stress gradient and cyclic strain hardening.

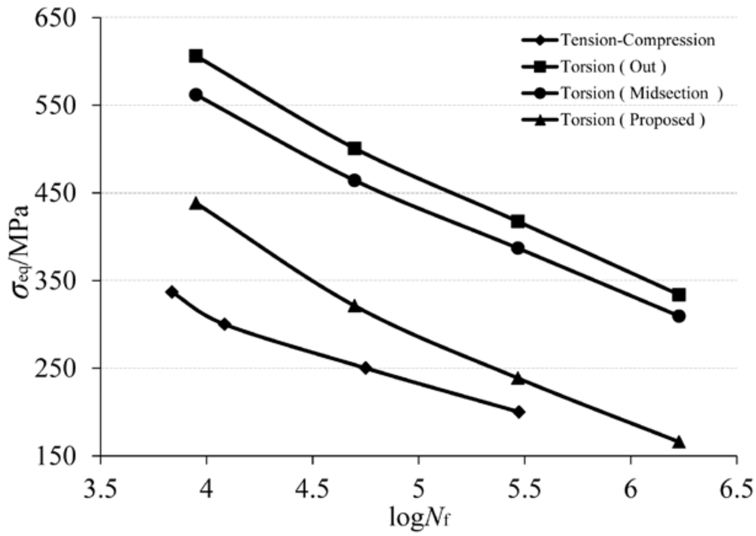

Figure 3 σ_{eq}-N_f curves of different shear stress parameters

5. Conclusion

- For a thin-walled tubular specimen under torsional loading, there is stress gradient on the cross section. This stress gradient leads to the fact that fatigue lifetime observed under fully reversed torsion is always higher than that under tension-compression for the same equivalent stress level.

- The newly proposed shear stress fatigue damage parameter takes the influences of both shear stress gradient and cyclic strain hardening into account. Compared with other two shear stress parameters, the $\sigma_{\mathrm{eq}} - N_{\mathrm{f}}$ curve translated from τ_{d} makes best coincide with the *S*-*N* curve in tension-compression, almost having eliminated the discrepancy.

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